

Cambridge International AS & A Level

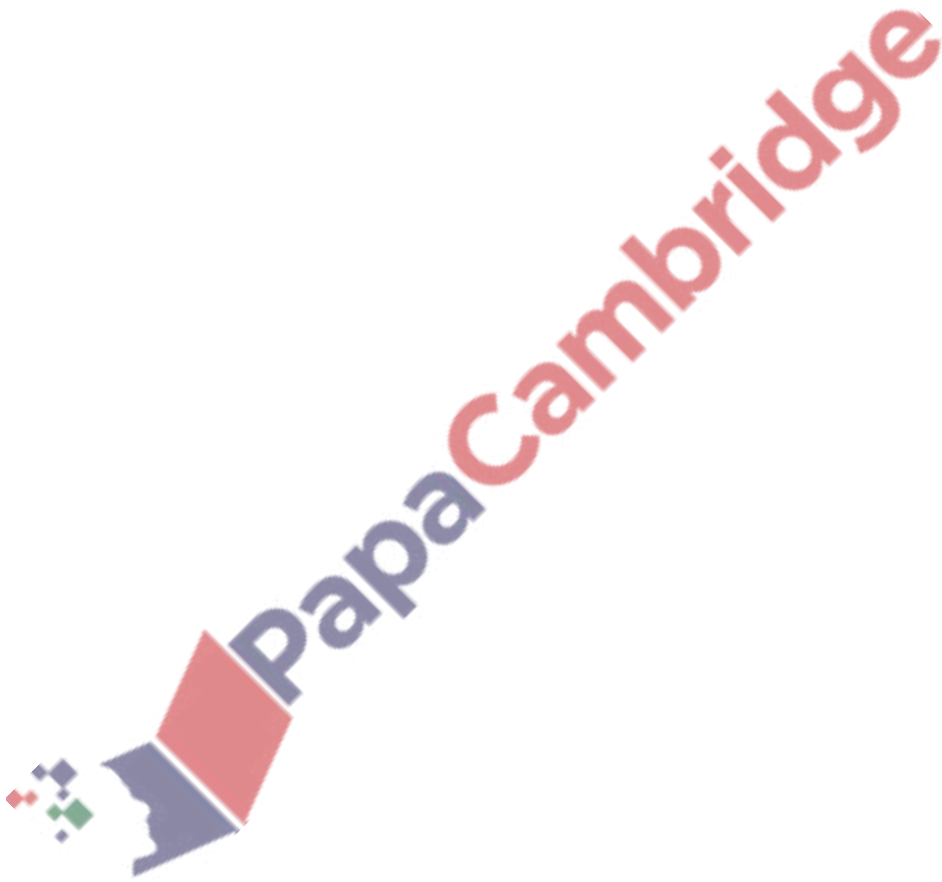
# MATHEMATICS (9709) P3

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS



## Chapter 7

# Vectors



248. 9709\_s20\_qp\_31 Q: 9

With respect to the origin  $O$ , the vertices of a triangle  $ABC$  have position vectors

$$\overrightarrow{OA} = 2\mathbf{i} + 5\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

- (a) Using a scalar product, show that angle  $ABC$  is a right angle. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Show that triangle  $ABC$  is isosceles. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



249. 9709\_s20\_qp\_32 Q: 10

With respect to the origin  $O$ , the points  $A$  and  $B$  have position vectors given by  $\vec{OA} = 6\mathbf{i} + 2\mathbf{j}$  and  $\vec{OB} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ . The midpoint of  $OA$  is  $M$ . The point  $N$  lying on  $AB$ , between  $A$  and  $B$ , is such that  $AN = 2NB$ .

- (a) Find a vector equation for the line through  $M$  and  $N$ . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The line through  $M$  and  $N$  intersects the line through  $O$  and  $B$  at the point  $P$ .

(b) Find the position vector of  $P$ .

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Calculate angle  $OPM$ , giving your answer in degrees.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



(b) Find angle  $BAD$ , giving your answer in degrees.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the area of the parallelogram correct to 3 significant figures.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....







252. 9709\_w20\_qp\_32 Q: 8

With respect to the origin  $O$ , the position vectors of the points  $A$ ,  $B$ ,  $C$  and  $D$  are given by

$$\vec{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \quad \vec{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \vec{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

- (a) Show that  $AB = 2CD$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the angle between the directions of  $\vec{AB}$  and  $\vec{CD}$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

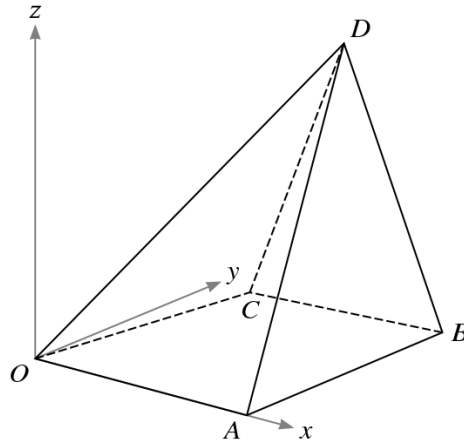
.....







254. 9709\_s19\_qp\_31 Q: 9



The diagram shows a set of rectangular axes  $Ox$ ,  $Oy$  and  $Oz$ , and four points  $A$ ,  $B$ ,  $C$  and  $D$  with position vectors  $\vec{OA} = 3\mathbf{i}$ ,  $\vec{OB} = 3\mathbf{i} + 4\mathbf{j}$ ,  $\vec{OC} = \mathbf{i} + 3\mathbf{j}$  and  $\vec{OD} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ .

- (i) Find the equation of the plane  $BCD$ , giving your answer in the form  $ax + by + cz = d$ . [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Calculate the acute angle between the planes  $BCD$  and  $OABC$ . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....









- (ii) It is given that  $l$  lies in the plane with equation  $ax + by + 2z = 13$ , where  $a$  and  $b$  are constants. Find the values of  $a$  and  $b$ . [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





258. 9709\_w19\_qp\_33 Q: 7

The plane  $m$  has equation  $x + 4y - 8z = 2$ . The plane  $n$  is parallel to  $m$  and passes through the point  $P$  with coordinates  $(5, 2, -2)$ .

- (i) Find the equation of  $n$ , giving your answer in the form  $ax + by + cz = d$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Calculate the perpendicular distance between  $m$  and  $n$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





259. 9709\_m18\_qp\_32 Q: 10

The line  $l$  has equation  $\mathbf{r} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ . The plane  $p$  has equation  $2x - 3y - z = 4$ .

- (i) Find the position vector of the point of intersection of  $l$  and  $p$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Find the acute angle between  $l$  and  $p$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....









.....

.....

.....

.....

.....

- (iii) Given that  $p$  is equidistant from the lines  $l$  and  $m$ , find the equation of  $p$ . Give your answer in the form  $ax + by + cz = d$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....







263. 9709\_w18\_qp\_31 Q: 10

The planes  $m$  and  $n$  have equations  $3x + y - 2z = 10$  and  $x - 2y + 2z = 5$  respectively. The line  $l$  has equation  $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ .

(i) Show that  $l$  is parallel to  $m$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Calculate the acute angle between the planes  $m$  and  $n$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





















- (ii) Find the equation of the plane through the origin which contains  $l$ . Give your answer in the form  $ax + by + cz = d$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Find the exact value of the perpendicular distance of  $A$  from this plane. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





269. 9709\_w17\_qp\_31 Q: 10

The equations of two lines  $l$  and  $m$  are  $\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$  and  $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$  respectively.

- (i) Show that the lines do not intersect. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Calculate the acute angle between the directions of the lines. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



270. 9709\_w17\_qp\_32 Q: 10

Two planes  $p$  and  $q$  have equations  $x + y + 3z = 8$  and  $2x - 2y + z = 3$  respectively.

- (i) Calculate the acute angle between the planes  $p$  and  $q$ . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) The point  $A$  on the line of intersection of  $p$  and  $q$  has  $y$ -coordinate equal to 2. Find the equation of the plane which contains the point  $A$  and is perpendicular to both the planes  $p$  and  $q$ . Give your answer in the form  $ax + by + cz = d$ . [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



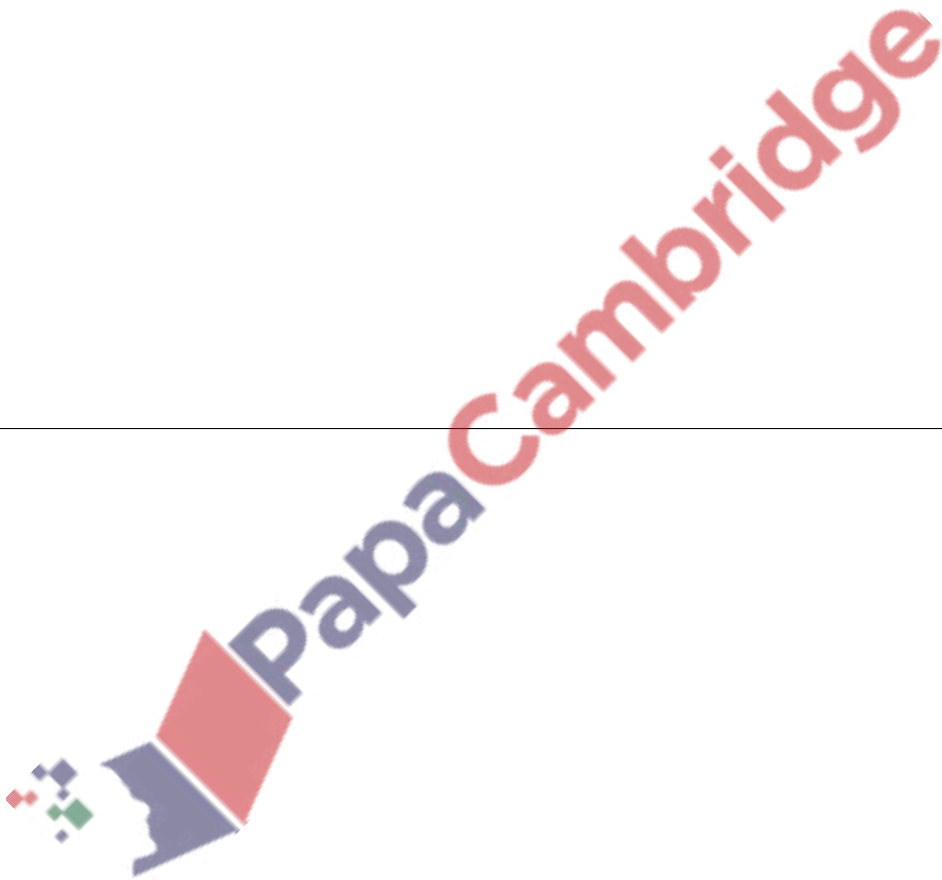


271. 9709\_m16\_qp\_32 Q: 8

The line  $l$  has equation  $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ . The plane  $p$  has equation  $\mathbf{r} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 6$ .

(i) Show that  $l$  is parallel to  $p$ . [3]

(ii) A line  $m$  lies in the plane  $p$  and is perpendicular to  $l$ . The line  $m$  passes through the point with coordinates  $(5, 3, 1)$ . Find a vector equation for  $m$ . [6]

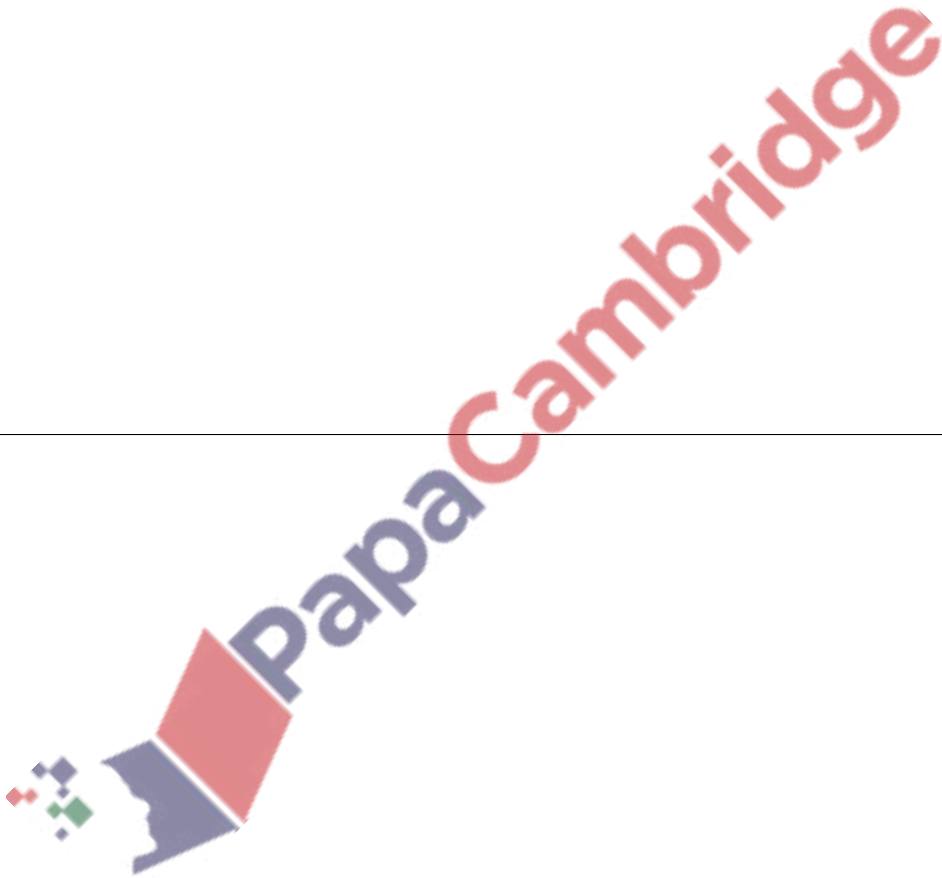


272. 9709\_s16\_qp\_31 Q: 9

With respect to the origin  $O$ , the points  $A, B, C, D$  have position vectors given by

$$\vec{OA} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}, \quad \vec{OB} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \vec{OC} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}, \quad \vec{OD} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

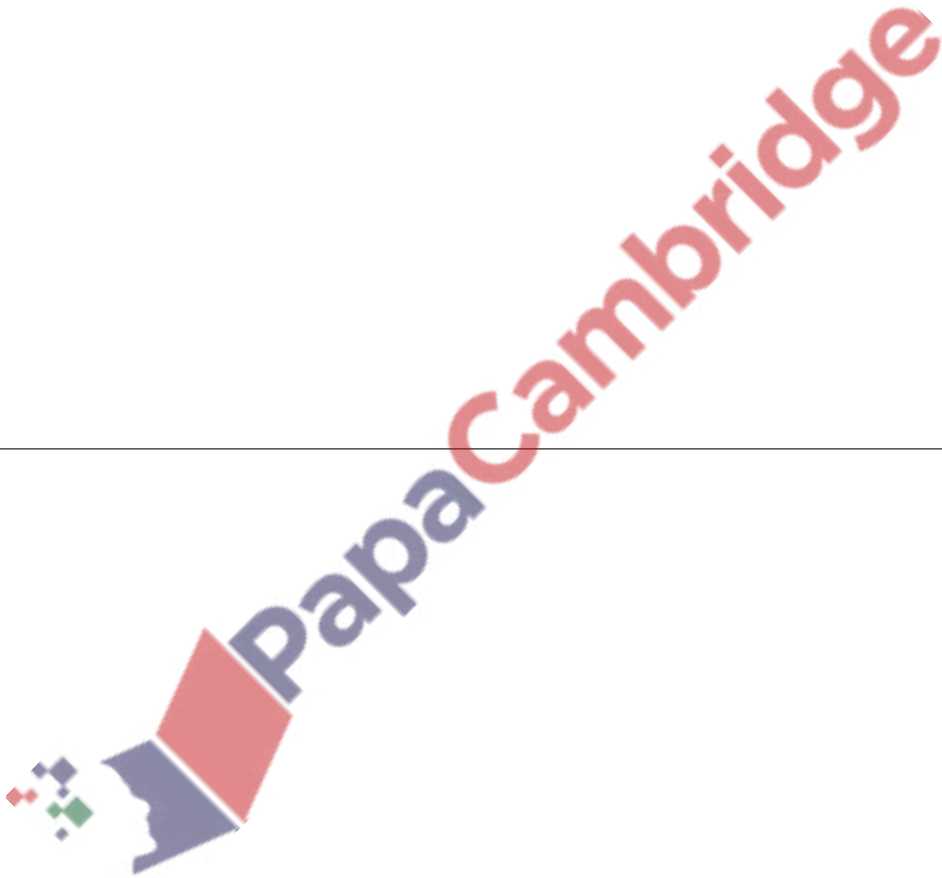
- (i) Find the equation of the plane containing  $A, B$  and  $C$ , giving your answer in the form  $ax + by + cz = d$ . [6]
- (ii) The line through  $D$  parallel to  $OA$  meets the plane with equation  $x + 2y - z = 7$  at the point  $P$ . Find the position vector of  $P$  and show that the length of  $DP$  is  $2\sqrt{14}$ . [5]



273. 9709\_s16\_qp\_32 Q: 9

The points  $A$ ,  $B$  and  $C$  have position vectors, relative to the origin  $O$ , given by  $\vec{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ ,  $\vec{OB} = 4\mathbf{j} + \mathbf{k}$  and  $\vec{OC} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ . A fourth point  $D$  is such that the quadrilateral  $ABCD$  is a parallelogram.

- (i) Find the position vector of  $D$  and verify that the parallelogram is a rhombus. [5]
- (ii) The plane  $p$  is parallel to  $OA$  and the line  $BC$  lies in  $p$ . Find the equation of  $p$ , giving your answer in the form  $ax + by + cz = d$ . [5]

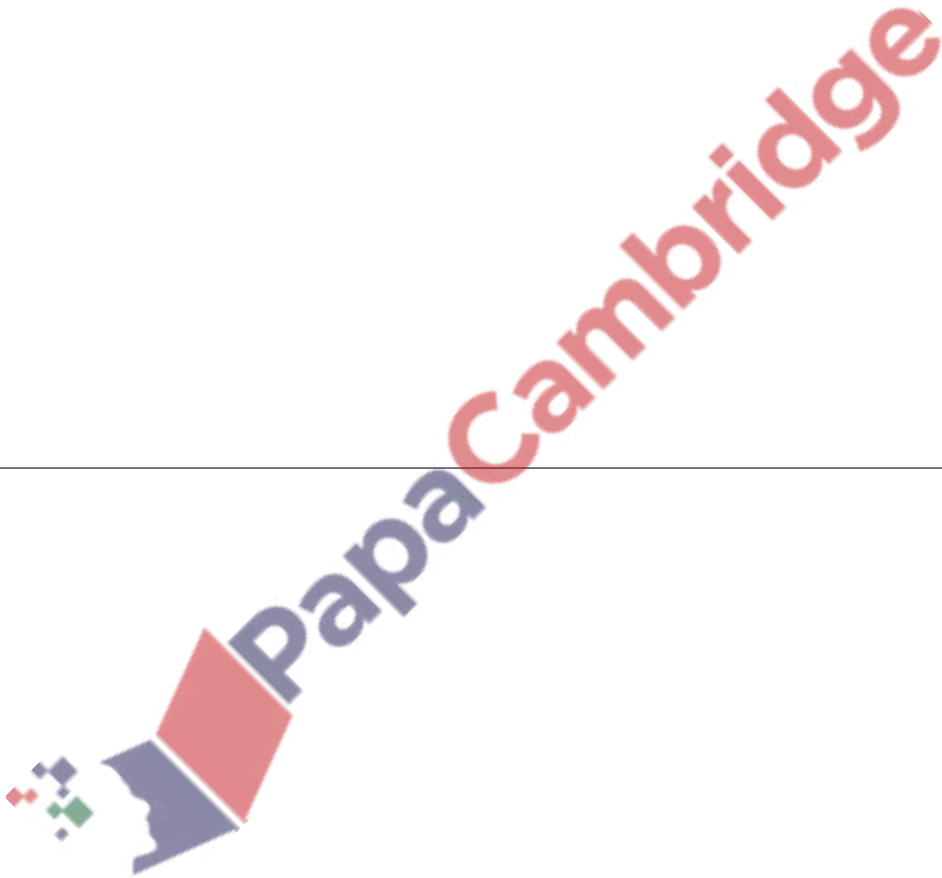


274. 9709\_s16\_qp\_33 Q: 8

The points  $A$  and  $B$  have position vectors, relative to the origin  $O$ , given by  $\vec{OA} = \mathbf{i} + \mathbf{j} + \mathbf{k}$  and  $\vec{OB} = 2\mathbf{i} + 3\mathbf{k}$ . The line  $l$  has vector equation  $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ .

(i) Show that the line passing through  $A$  and  $B$  does not intersect  $l$ . [4]

(ii) Show that the length of the perpendicular from  $A$  to  $l$  is  $\frac{1}{\sqrt{2}}$ . [5]

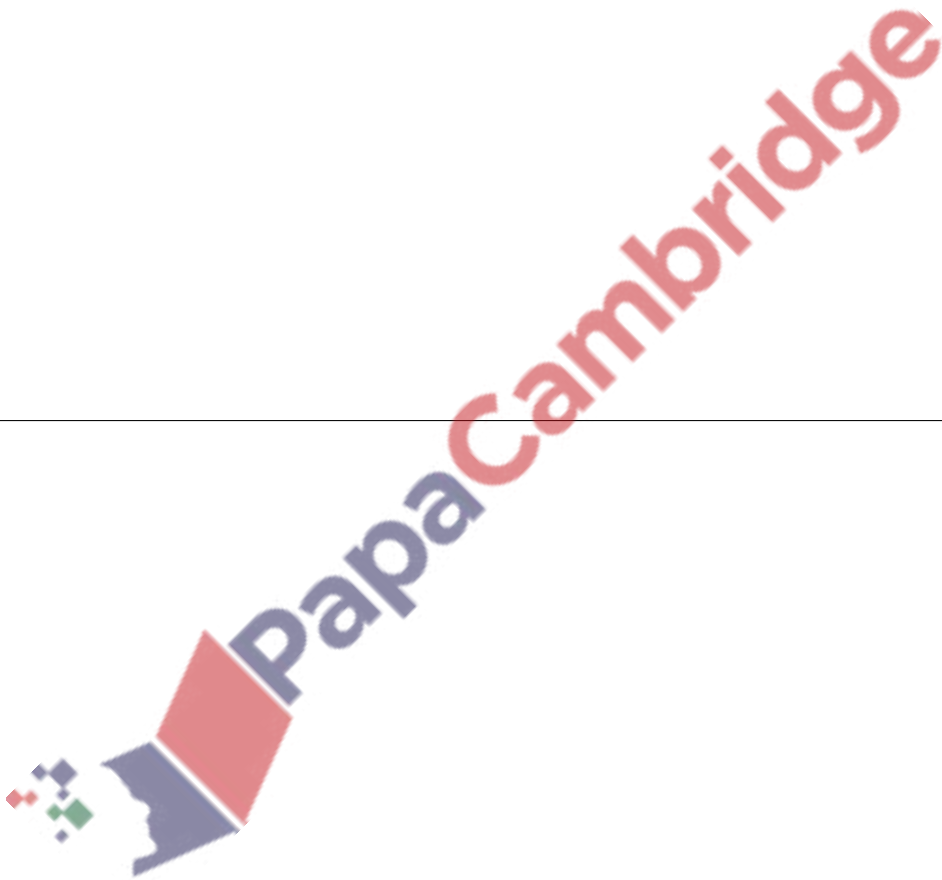


275. 9709\_w16\_qp\_31 Q: 8

Two planes have equations  $3x + y - z = 2$  and  $x - y + 2z = 3$ .

- (i) Show that the planes are perpendicular. [3]
- (ii) Find a vector equation for the line of intersection of the two planes. [6]

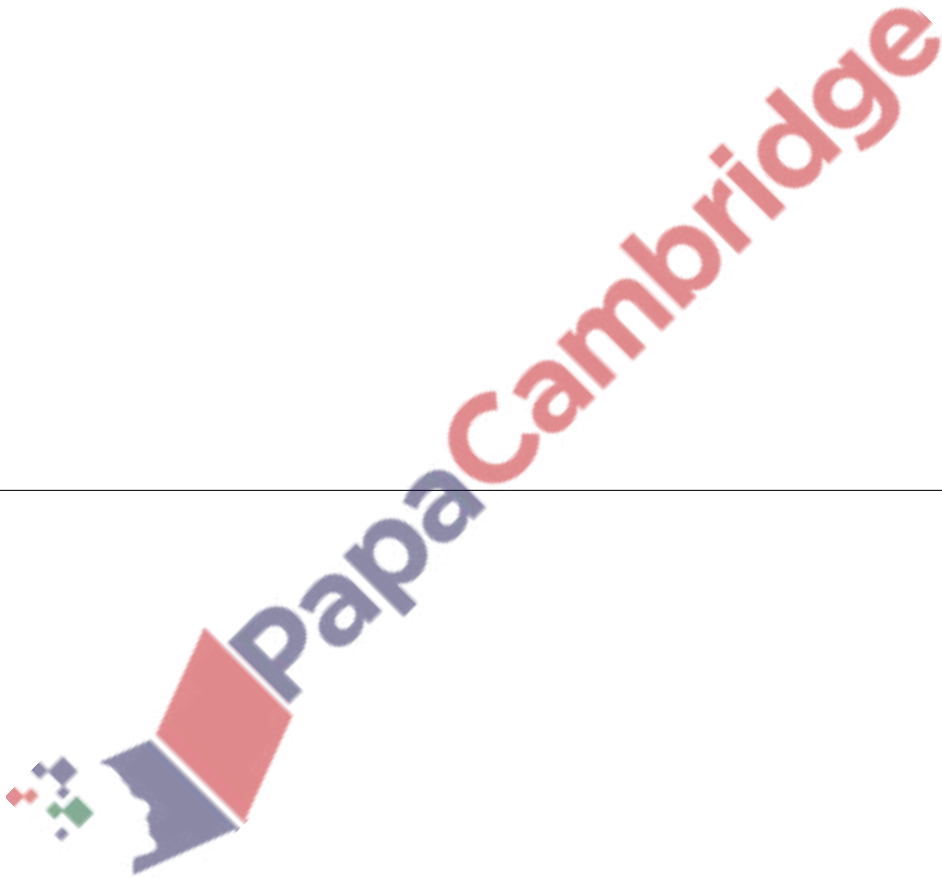
---

 PapaCambridge

276. 9709\_w16\_qp\_33 Q: 10

The line  $l$  has vector equation  $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ .

- (i) Find the position vectors of the two points on the line whose distance from the origin is  $\sqrt{10}$ . [5]
- (ii) The plane  $p$  has equation  $ax + y + z = 5$ , where  $a$  is a constant. The acute angle between the line  $l$  and the plane  $p$  is equal to  $\sin^{-1}(\frac{2}{3})$ . Find the possible values of  $a$ . [5]



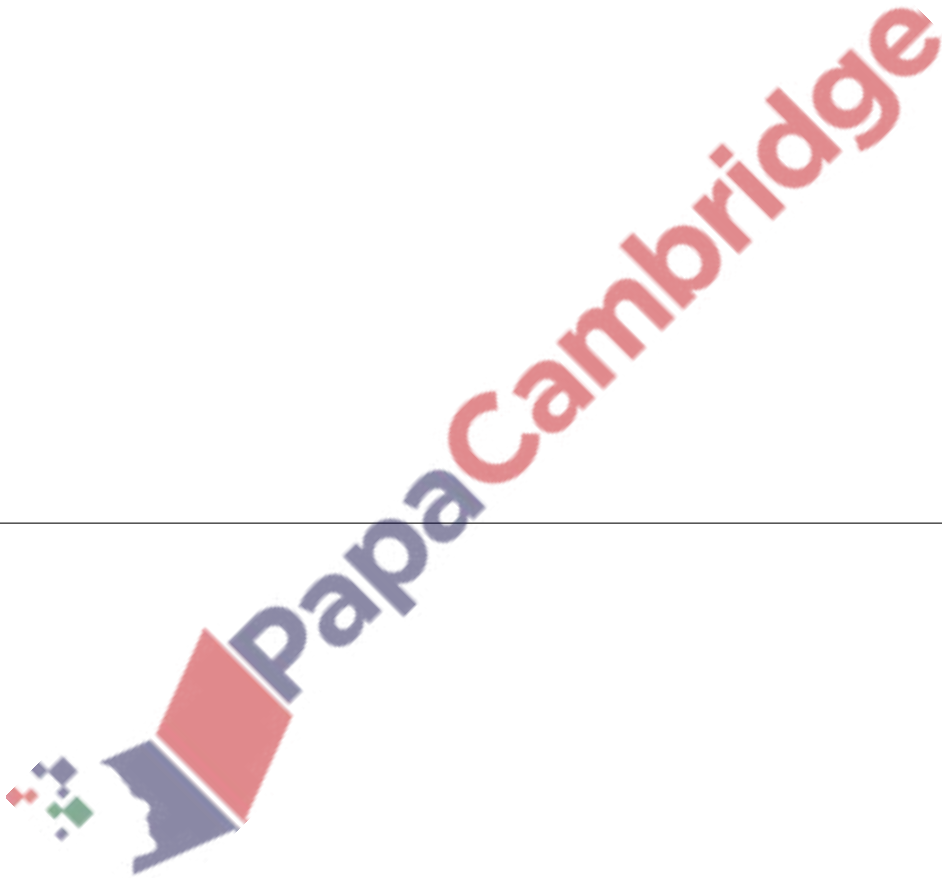
277. 9709\_s15\_qp\_31 Q: 6

The straight line  $l_1$  passes through the points  $(0, 1, 5)$  and  $(2, -2, 1)$ . The straight line  $l_2$  has equation  $\mathbf{r} = 7\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$ .

(i) Show that the lines  $l_1$  and  $l_2$  are skew. [6]

(ii) Find the acute angle between the direction of the line  $l_2$  and the direction of the  $x$ -axis. [3]

---

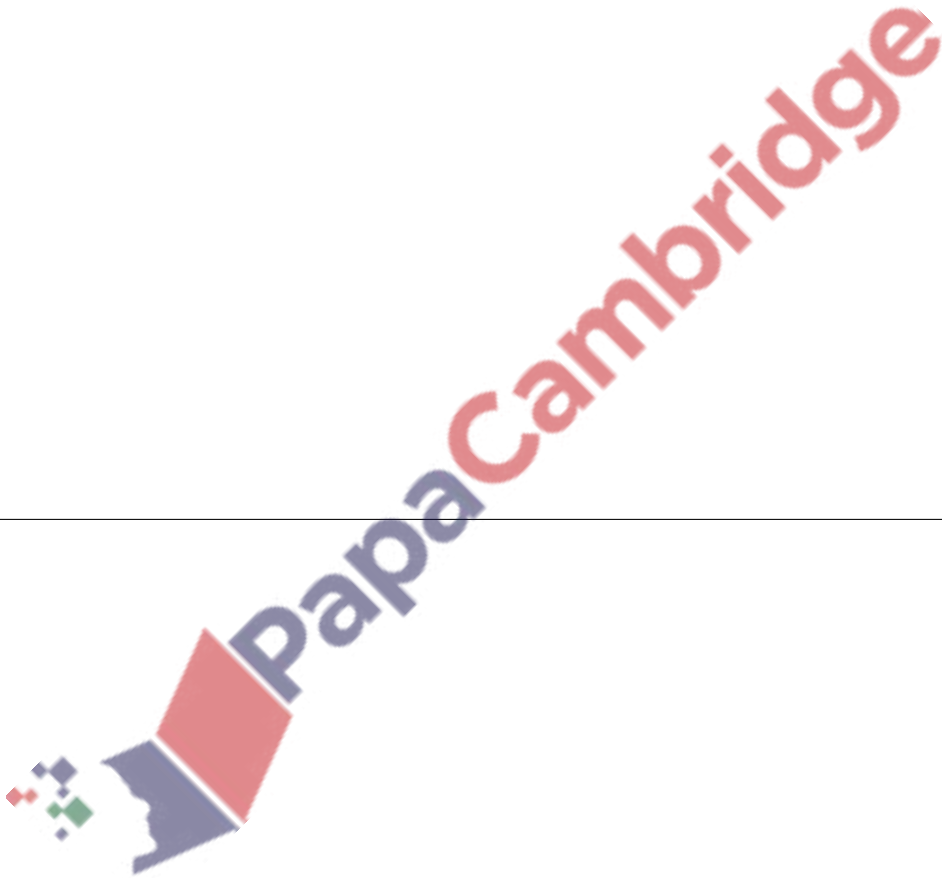




278. 9709\_s15\_qp\_32 Q: 10

The points  $A$  and  $B$  have position vectors given by  $\vec{OA} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$  and  $\vec{OB} = \mathbf{i} + \mathbf{j} + 5\mathbf{k}$ . The line  $l$  has equation  $\mathbf{r} = \mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ .

- (i) Show that  $l$  does not intersect the line passing through  $A$  and  $B$ . [5]
- (ii) Find the equation of the plane containing the line  $l$  and the point  $A$ . Give your answer in the form  $ax + by + cz = d$ . [6]



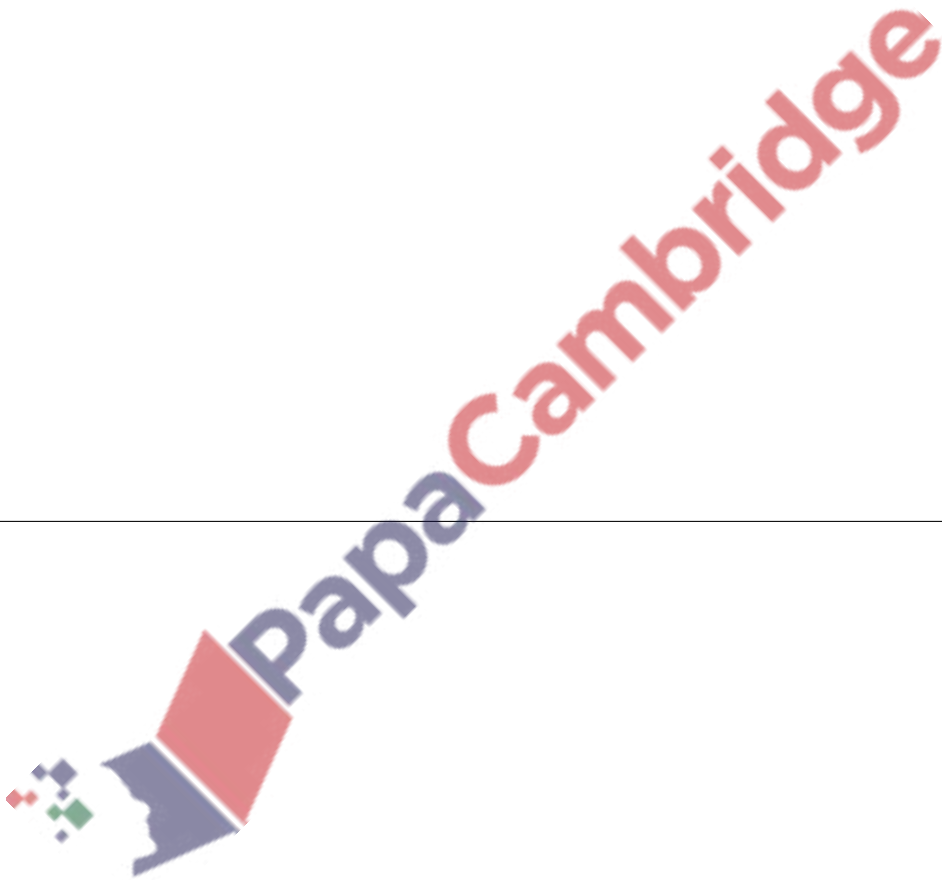
279. 9709\_s15\_qp\_33 Q: 9

Two planes have equations  $x + 3y - 2z = 4$  and  $2x + y + 3z = 5$ . The planes intersect in the straight line  $l$ .

(i) Calculate the acute angle between the two planes. [4]

(ii) Find a vector equation for the line  $l$ . [6]

---

 PapaCambridge

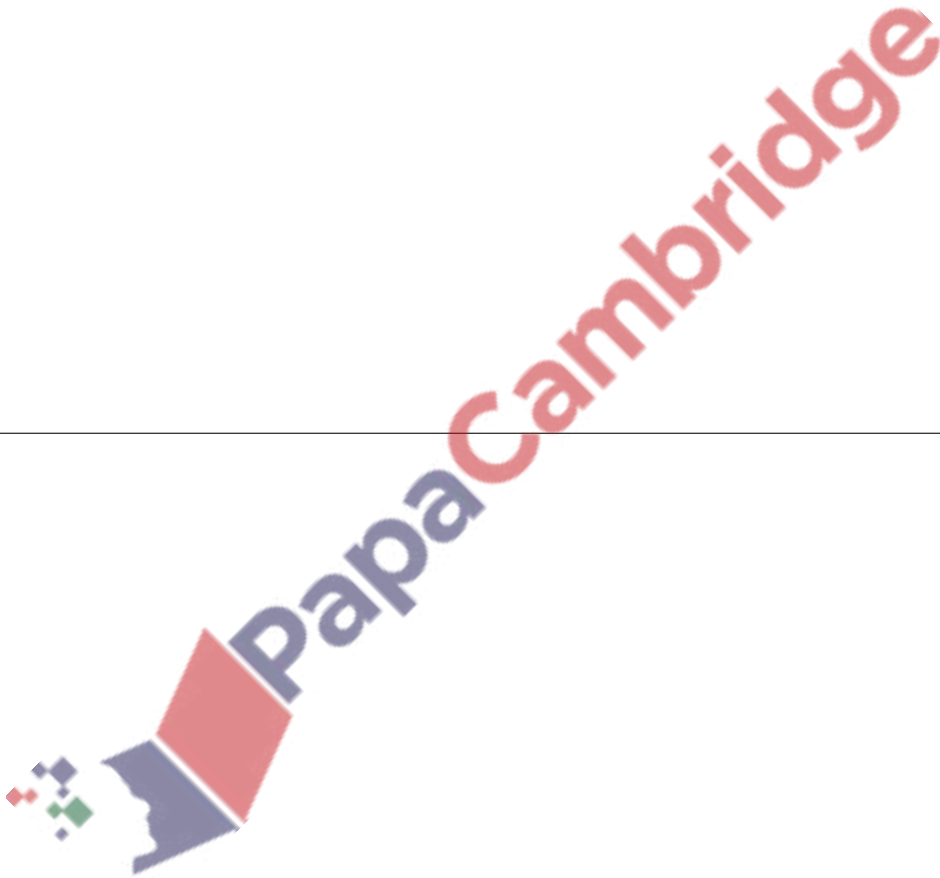
280. 9709\_w15\_qp\_31 Q: 7

The points  $A$ ,  $B$  and  $C$  have position vectors, relative to the origin  $O$ , given by

$$\vec{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}.$$

The plane  $m$  is perpendicular to  $AB$  and contains the point  $C$ .

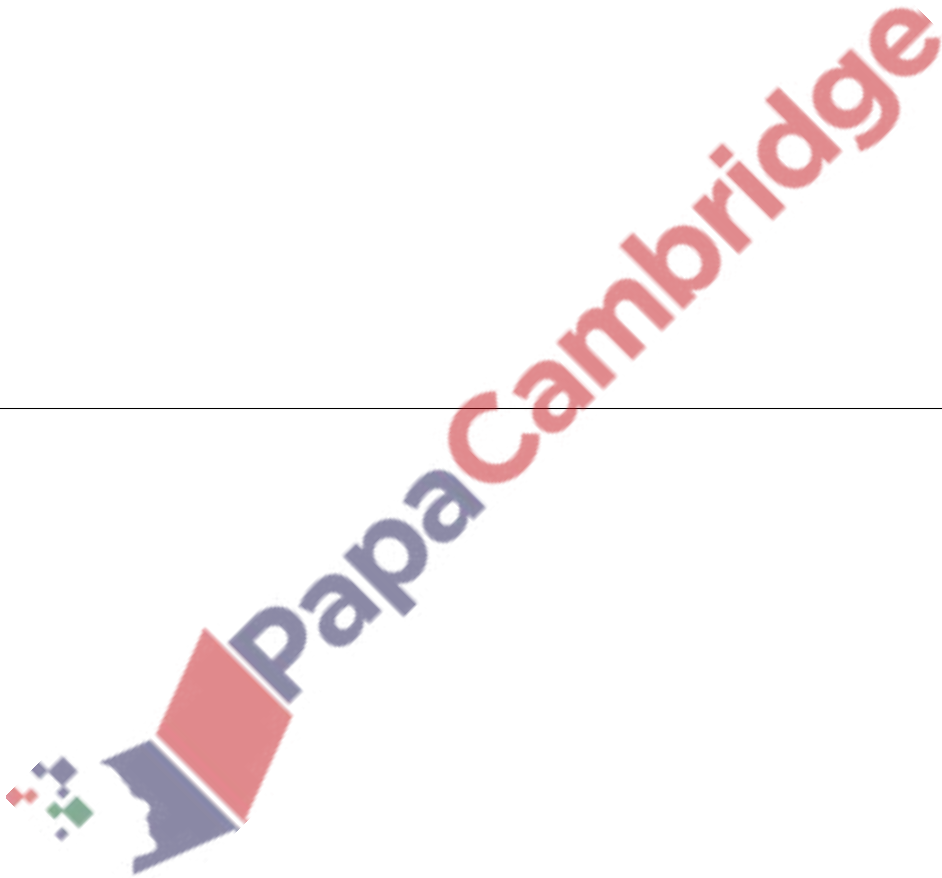
- (i) Find a vector equation for the line passing through  $A$  and  $B$ . [2]
- (ii) Obtain the equation of the plane  $m$ , giving your answer in the form  $ax + by + cz = d$ . [2]
- (iii) The line through  $A$  and  $B$  intersects the plane  $m$  at the point  $N$ . Find the position vector of  $N$  and show that  $CN = \sqrt{13}$ . [5]




281. 9709\_w15\_qp\_33 Q: 8

A plane has equation  $4x - y + 5z = 39$ . A straight line is parallel to the vector  $\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$  and passes through the point  $A(0, 2, -8)$ . The line meets the plane at the point  $B$ .

- (i) Find the coordinates of  $B$ . [3]
- (ii) Find the acute angle between the line and the plane. [4]
- (iii) The point  $C$  lies on the line and is such that the distance between  $C$  and  $B$  is twice the distance between  $A$  and  $B$ . Find the coordinates of each of the possible positions of the point  $C$ . [3]



 **PapaCambridge**